Chapter 1: Patterns and Recursion p. 27 9/26

1.1 Recursively Defined Sequences

Sequence - the order things are in

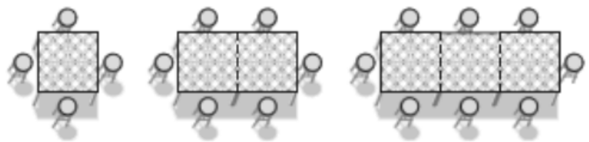
Ex. A
p. 28

$4\ u_1 \quad 6\ u_2 \quad 8\ u_3 \quad 10\ u_4 \quad 12\ u_5 \quad \ldots \quad u_6$

$+2 \quad +2 \quad +2 \quad +2 \quad +2$

$n=6$

term - part of a sequence



recursive formula

$u_6 = u_5 + 2$

$= 12 + 2 = 14$

$u_1 = 4$

$n = 2$ second term

n

$u_n = u_{n-1} + 2$

where $n \geq 2$

u_n = next term

u_{n-1} = previous term

Ex. B
p. 30

1	2	3	4
59	63	67	

$u_1 = 59$

$u_n = u_{n-1} + 4$

Where $n \geq 2$

$u_4 = u_3 + 4 = 71$

$u_n = 95 = u_{n-1} + 4$

$67 + 4(n-3) = 95$

$67 + 4n - 12 = 95$

$4n + 55 = 95$

$59 + 4(n-1) = 95$

$59 + 4n - 4 = 95$

$4n + 55 = 95$

$-55 \quad -55$

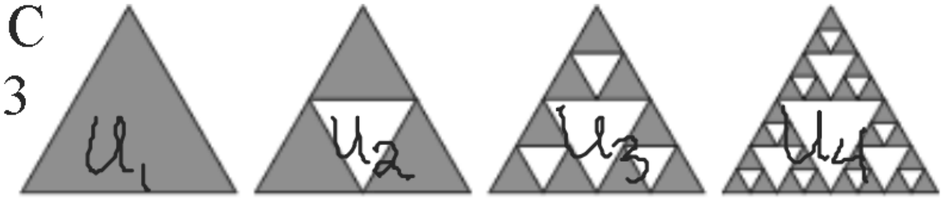
$\frac{4n}{4} = \frac{40}{4}$

$n = 10$

Arithmetic

Arithmetic Sequence - a sequence in which each term is equal to the previous term plus (or minus) a constant. This constant is called the common difference. $u_n = u_{n-1} + d$

Ex. C
p. 33



1 $\times 3$ 3 $\times 3$ 9 $\times 3$ 27

$u_1 = 1$

$u_n = u_{n-1} \times 3$

where $n \geq 2$

HW p.34
#1–6

Geometric Sequence - a sequence in which each term is equal to the previous term multiplied by a constant. This constant is called the common ratio. $u_n = r \cdot u_{n-1}$

1c) $u_0 = 32$ 32, 48, 72, 108

$u_n = 1.5\,u_{n-1}$

where $n \geq 1$

2c) geometric, $r = 1.5$

4) arithmetic sequence $d = 3.2$, $u_1 = 6$

$u_1 = 6$

$u_n = u_{n-1} + 3.2$

where $n \geq 2$

$u_{10} = 6 + 3.2(n-1)$

$10-1 = 9$

$= 6 + 3.2(9) = 6 + 28.8$

$u_{10} = 34.8$

5a)

$u_{15} = 2 + 4(15-1)$

$u_n = u_1 + d(n-1)$

5c) 0.4, 0.04, 0.004, 0.0004

$u_1 = 0.4$

$u_n = 0.1\, u_{n-1}$

where $n \geq 2$

4 E-10

4×10^{-10}

0.0000000004

Arithmetic sequence $u_n = u_{n-1} + d$

Geometric sequence $u_n = r * u_{n-1}$

$r > 1$ growth

$r < 1$ decay

Ex. A depreciation

$u_0 = \$23,999$

$-\frac{1}{5}$ each year

Value of car at 6yr old

Lose $\frac{1}{5}$, Keep $\frac{4}{5}$

$u_n = 0.8\, u_{n-1}$

where $n \geq 1$

23999 enter

x0.8 enter

u_1

$u_6 = \$6,291.19$

Ex. B Interest $2,000 7% compounded annually

no W/D Value of initial deposit doubles

($4,000)

$u_0 = 2000$

$u_n = (1 + .07)u_{n-1}$

where $n \geq 1$

2,000 enter

$\times 1.07$ enter

u_1

$u_n \geq 4{,}000$

$u_{11} = \$4{,}209.70$

1.003

Simple interest

208 $\times r$

pp. 41-42 #1, 2, 3, 8, 9

3b) $u_1 = 73.4375$

$u_n = 0.4\, u_{n-1}$

where $n \geq 2$

$\underbrace{.025}$

2.5% Annual Interest Rate

9a) 12, 14.4   14, 16.8

2   3

17, 20.4   21, 25.2   25, 30

4   4   5

2, 3, 4, 4, 5

b) 30

Limit – the quantity associated with the stability of a system

can use long run values to approximate the limit

Shifted geometric sequence – includes an added term

$u_0 =$

$u_n = r u_{n-1} + d$

where $n \geq 1$

Ex. 450g Cl shock treatment

add 45g/day lose 15% per day

Day 1, Day 2, Day 3, long run value

$U_0 = 450$

$U_n = (1 - .15)U_{n-1} + 45$

where $n \geq 1$

$U_1 = (0.85)450 + 45 = 427.5$

$U_2 = (0.85)427.5 + 45 = 408.4$

$U_3 = 392.14$

$\frac{45}{.15} = \frac{.15}{.15}C \quad C = 300g$

HW p.48 #1-4

2a) $a = 210 + 0.75a$

$-0.75a \qquad -0.75a$

$$\frac{0.25a}{0.25} = \frac{210}{0.25}$$

$$a = 840$$

d) $d = 0.75d$

$-0.75d \quad -0.75d$

$$\frac{0.25d}{0.25} = \frac{0}{0.25}$$

$$d = 0$$

4b) 0, 10, 15, 17.5, 18.75

$U_1 = 0$

$U_n = \underline{0.5}\, U_{n-1} + 10$

Where $n \geq 2$

3 a) $u_0 = 16$

$u_n = (1 - 0.05)u_{n-1} + 16$

$$\frac{0.05u}{0.05} = \frac{16}{0.05}$$

$$u = 320$$

c) $u_n = (1 - 0.10)u_{n-1}$

$$0 = .1u$$

$$0 = u$$

1.4 Graphing Sequences p. 51 10/4

Inv.

Table	Recur. form.	Graph
1	B	IV
2	C	VI
3	F	III
4	D	V
5	A	II
6	E	I

arithmetic graphs - linear

geometric graphs -
non-linear
zero limit or no limit

shifted geometric graphs -
non-linear
non-zero limit
zero limit possible

Continuous

discrete

pp. 54–56
#1, 2, 3, 5

1 a)

	min	max
n	0	9
U_n	0	16

b)

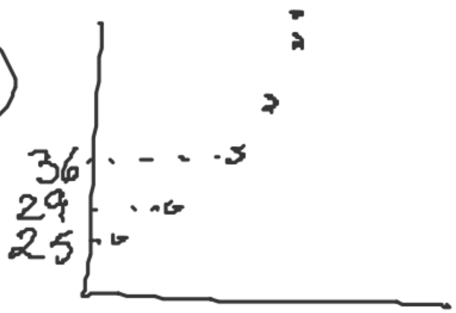
	min	max
n	0	20
U_n	0	400

c)

	min	max
n	0	30
U_n	200	25

d)

	min	max
n	0	70
U_n	15	3300

2a) 

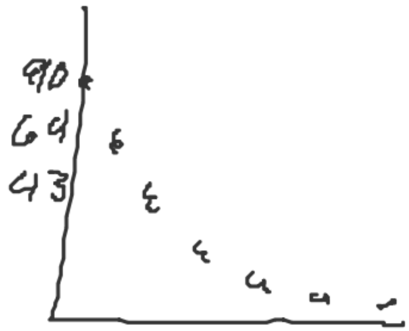
geometric

$$\frac{36}{29} = 1.24$$

$$\frac{29}{25} = 1.16$$

$$U_n = 1.2\,U_{n-1}$$

$$U_n = (1+0.2)\,U_{n-1}$$

2d) 

$$U_0 = 90$$

$$U_n = 0.69\,U_{n-1}$$

$$\frac{64}{90} = 0.71$$

$$\frac{43}{64} = .67$$

Note 1D • Sequence Mode

Sequence mode is a powerful way of working with recursive formulas. Press $\boxed{\text{MODE}}$, scroll down to the fourth line, and select Seq. Then go to the Y= screen.

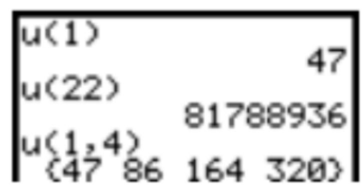
Follow these steps to enter the recursive formula

$u_1 = 47$
$u_n = 2u_{n-1} - 8 \quad \text{where } n \geq 2$

a. Set *n*Min to be the *n*-value of the starting term; in this example enter 1.

b. Enter the equation for u(*n*)=. To get $u(n-1)$ press $\boxed{\text{2nd}}$ [u] $\boxed{(}$ $\boxed{\text{X,T,θ,n}}$ $\boxed{-}$ $\boxed{1}$ $\boxed{)}$.

c. Set u(*n*Min) to be the value of the starting term; in this example enter 47. (The calculator will put the value in braces.)



You can find values of individual terms, as well as a range of terms, on the Home screen. To find u_{22}, press $\boxed{\text{2nd}}$ [u] (22). To find a range of terms, use a comma between the first and last term.

Note 1E • Graphing Sequence Mode

You can graph sequences to display numbers generated by recursive formulas. The x-axis will represent the values of n, and the y-axis will represent the values of $u(n)$.

Go to the Window screen. Set the window values to show the part of the graph you want to see.

nMin = the smallest value of n you want graphed on the x-axis. You've already set this on the Y= screen.

nMax = a value a little larger than the greatest value of n you want graphed.

PlotStart = the first term of the sequence you want graphed. This is almost always 1.

PlotStep = the terms you want graphed. For example, if you want to plot every other term, PlotStep=2. PlotStep is almost always 1.

Xmin = and Xmax = the minimum and maximum values on the x-axis. These usually will be about the same as nMin and nMax, unless you want a close-up look at some part of the graph.

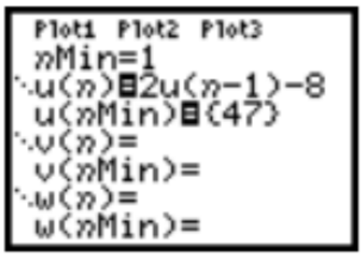
Xscl = and Yscl = the distance between tick marks on the two axes. The number of divisions should be less than 25. If there are too many tick marks, the axes will appear too thick.

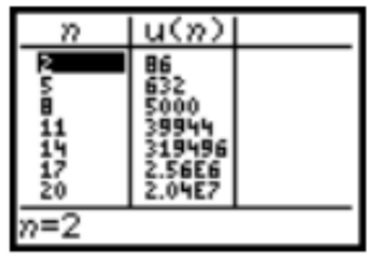
Ymin = and Ymax = the range of function values you want graphed. Usually Ymin will be slightly less than the smallest function value and Ymax will be slightly greater than the largest function value.

Press GRAPH to see the graph.

Note 1K • Sequence Tables

You can view many elements of a sequence at once by using sequence tables. First enter the sequence into the Y= screen. (See **Note 1D** if you need help entering a sequence.) Then press $\boxed{\text{2nd}}$ [TBLSET]. TblStart is the smallest *n*-value for which you wish to see a sequence value. The value of ΔTbl specifies which terms will actually be displayed. For example, if ΔTbl=3 the table will display every third term. Press $\boxed{\text{2nd}}$ [TABLE] to display the table. Use the up and down arrow keys to see more *x*-values, or the right and left arrow keys to see values of other sequences that are entered.





\$22,000 7.9% annually

5yr (60 mo)

monthly payments

$u_0 = 22{,}000$

$u_n = \left(1 + \frac{0.079}{12}\right) u_{n-1} - 300$

where $n \geq 1$

	1
mo	12
quarterly	4
daily	365

$(1.00658\overline{3})u_{n-1} - 300$

2nd $u(n-1) - 300$

$u_{61} = 0$

59 pmnts \$439.08 last pmnt \$438.50

Ex. Investment AIR 4.75% C. Weekly

contrib $10/wk 5yr (260 wk)

$u_0 = 10$

$u_n = \left(1 + \frac{0.0475}{52}\right) u_{n-1} + 10$

$(1.000913) u_{n-1} + 10$

$u_{260} = \$2945.71$

p. 64 # 1-6a

1) $u_0 = 450$
$u_n = (1 + 0.039) u_{n-1} + 50$
where $n \geq 1$ n is years

Investment because the payment is added.

Principle is 450

Pmt/Dep 50

Int. 3.9%

Annually (divide by 1)

0.04

4%

3) 32,000 find 1st mo. Int.

$\overline{12}$

a) $\frac{0.049}{12}(32,000) = \$130.67$

$(1 + (0.049/12))U_{n-1} + 100$

Chapter 1 Review

10/10

p. 71-73 # 1 - 6, 8, 9

2. 3, 7, 11, 15

a) arithmetic

b) $u_1 = 3+4(128-1)$
where ≥ 2

c) $u_{128} = 511$

d) The 40 term

e) $20 = 79$

6) $u_1 = 88$ gallons

$u_1 = 3$
$u_n = u_{n-1} + 4$

1. a geometric

b. $u_1 = 256$
$u_n = u_{n-1} \times .75$
where ≥ 2

c. 34.17

D. 10^{th}

e. 2.57

a)

$U_1 = -3$

$U_n = U_{n-1} + 1.5$

where $n \geq 2$

$-3, -1.5, 0, 1.5, 3$

min = 1 max = 5

$U_n = -3 - 3$

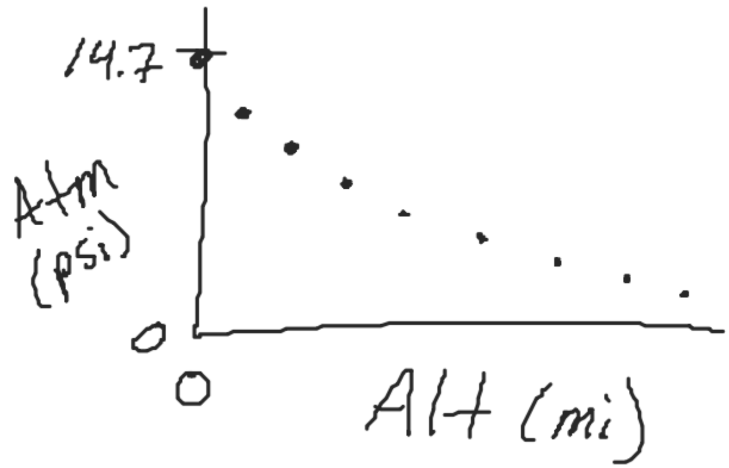
2, 4, 10, 28, 82

n	1	5
U_n	2	82

$u_0 = 14.7$

$u_n = (1 - 0.20)u_{n-1}$

where $n \geq 1$



$u_7 = 3.08 \frac{lb}{in^2}$

$u_{18} = 12.4$ gal

+ 4.2 per min (18 min)

---

88 gal

$u_0 = 500$

$u_n = \left(1 + \frac{.055}{4}\right) u_{n-1}$

$u_{20} = \$657.05$

$u_{1970} = 34$

$u_n = (1 + 0.48) u_{n-1}$

$u_0 = 500$

$u_n = \left(1 + \frac{0.055}{4}\right) u_{n-1} + 150$

$u_{20} = \$4000.83 \frac{21}{100}$

$\$4083.21$

$$\begin{array}{r} 1.2 \\ 1.57 \\ 1.39 \\ 1.51 \\ +\ 1.43 \\ \hline 5.90 \end{array} /4$$

1.48